

Class -11 Autumn Break Home work

**Biology**

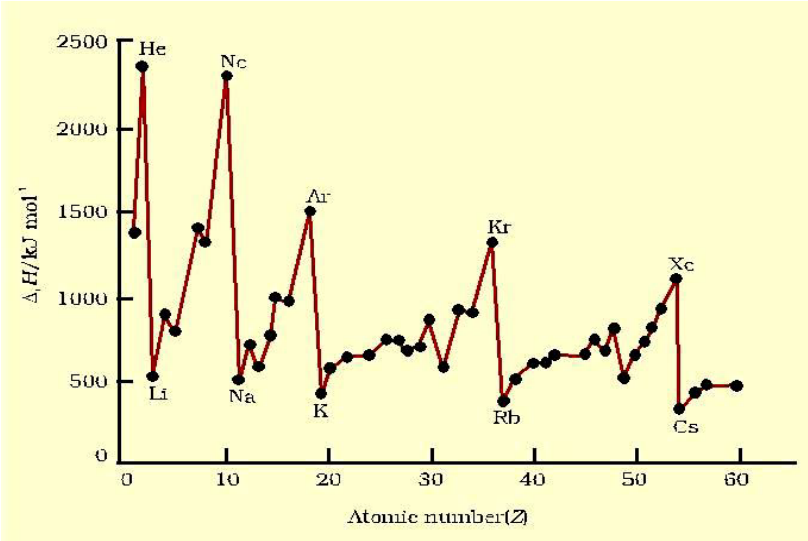
All the work to be done in classwork notebook

Unit – 1: Diversity in living world

1. Read and revise each chapter in given unit.
2. Prepare 5 question from each chapter.
3. Practice diagrams given in NCERT

Unit – 2: Structural Organisation in plant and animal

1. Read and revise each chapter in given unit.
2. Prepare 5 question from each chapter.
3. Practice diagrams given in NCERT
4. Solve the practice test paper of half yearly provided in the class.

1.	<p>The enthalpies of all elements in their standard states are</p> <p>(a) unity  (b) zero  (c) &lt;0  (d) different for each element.</p>	
2.	<p>Discuss the shape of the following molecules using the VSEPR model:  SiCl<sub>4</sub>, AsF<sub>5</sub>, ClF<sub>3</sub>, NH<sub>3</sub></p>	
3.	<p>Ionization enthalpy and atomic radius are closely related properties. Analyze the following graph :</p>  <p>What conclusion can you derive from the graph regarding the first ionization enthalpies of alkali metals and noble gases? Justify your answer.</p>	
4.	<p>(i) The energy associated with first orbit in hydrogen atom is <math>-2.17 \times 10^{-18} \text{ J atom}^{-1}</math>. What is the energy associated with the fifth orbit ?  (ii) Calculate the radius of Bohr's fifth orbit for hydrogen atom.</p>	
5.	<p>i. Give the criteria for spontaneity of a process in terms of free energy change (<math>\Delta G</math>).  ii. Exothermic reactions associated with a decrease in entropy are spontaneous at lower temperatures. Justify on the basis of Gibbs equation.  iii. For the reaction <math>2 \text{ A(g)} + \text{ B(g)} \rightarrow 2 \text{ D(g)}</math> <math>\Delta U^0 = -10.5 \text{ kJ}</math> and <math>\Delta S^0 = -44.1 \text{ JK}^{-1}</math>. Calculate <math>\Delta G^0</math> for the reaction, and predict whether the reaction may occur spontaneously.</p>	
6.	<p>a. For a reaction both <math>\Delta H</math> and <math>\Delta S</math> are negative. Under what conditions does the reaction occur spontaneously?  b. Define- Lattice enthalpy  c.</p>	

	<p>Calculate the enthalpy change for the process</p> $\text{CCl}_4(\text{g}) \longrightarrow \text{C}(\text{g}) + 4 \text{Cl}(\text{g})$ <p>and calculate bond enthalpy of C – Cl in <math>\text{CCl}_4(\text{g})</math></p> <p>Given: <math>\Delta_{\text{vap}} H^\ominus (\text{CCl}_4) = 30.5 \text{ kJ mol}^{-1}</math>; <math>\Delta_f H^\ominus (\text{CCl}_4) = -135.5 \text{ kJ mol}^{-1}</math>  <math>\Delta_a H^\ominus (\text{C}) = 715.0 \text{ kJ mol}^{-1}</math> where <math>\Delta_a H^\ominus</math> is enthalpy of atomisation  <math>\Delta_a H^\ominus (\text{Cl}_2) = 242 \text{ kJ mol}^{-1}</math>.</p>	
7.	<p>The enthalpy of combustion of methane, graphite and dihydrogen at 298 K are, -890.3 kJ mol<sup>-1</sup>, -393.5 kJ mol<sup>-1</sup> and -285.8 kJ mol<sup>-1</sup> respectively. Enthalpy of formation of CH<sub>4</sub>(g) will be</p> <p>(a) -74.8 kJ mol<sup>-1</sup>  (b) -52.27 kJ mol<sup>-1</sup>  (c) +74.8 kJ mol<sup>-1</sup>  (d) +52.26 kJ mol<sup>-1</sup></p>	
8.	<p>The reaction of cyanamide, NH<sub>2</sub>CN(g), with dioxygen was carried out in a bomb calorimeter, and <math>\Delta U</math> was found to be -742.7 kJ mol<sup>-1</sup> at 298 K. Calculate enthalpy change for the reaction at 298 K.</p> $\text{NH}_2\text{CN}(\text{g}) + 3/2 \text{O}_2(\text{g}) \rightarrow \text{N}_2 + \text{CO}_2(\text{g}) + \text{H}_2\text{O}(\text{l})$	
9.	<p>Calculate the number of kJ of heat necessary to raise the temperature of 60.0 g of aluminium from 35°C to 55°C. Molar heat capacity of Al is 24 J mol<sup>-1</sup> K<sup>-1</sup>.</p>	
10.	<p>Molecular orbitals are formed by the overlap of atomic orbitals. Two atomic orbitals combine to form two molecular orbitals called bonding molecular orbital (BMO) and anti bonding molecular orbital (ABMO). Energy of anti bonding orbital is raised above the parent atomic orbitals that have combined and the energy of the bonding orbital is lowered than the parent atomic orbitals. Energies of various molecular orbitals for elements hydrogen to nitrogen increase in the order : Different atomic orbitals of one atom combine with those atomic orbitals of the second atom which have comparable energies and proper orientation. Further, if the overlapping is head on, the molecular orbital is called 'Sigma', (<math>\sigma</math>) and if the overlap is lateral, the molecular orbital is called 'pi', (<math>\pi</math>). The molecular orbitals are filled with electrons according to the same rules as followed for filling of atomic orbitals. However, the order for filling is not the same for all molecules or their ions. Bond order is one of the most important parameters to compare the strength of bonds.</p> <p>Answer the following .</p> <p>a. Calculate the bond order for Hydrogen molecule.  b. Calculate the bond order for He  c. Calculate the bond order and magnetic nature of Oxygen molecule</p>	

**AUTUMN BREAK HOME WORK CLASS XI –A/B SUBJECT IP / CS (2023-24)**

**COMPLETION OF PRACTICAL FILE**

1. Program to find simple interest
2. Program to check whether a person is eligible to vote or not when age is input by user.
3. Program to check a number is even or odd
4. Program to check a number is a negative, positive or zero.
5. Program to accept marks of a student and print his /her result (fail, 3<sup>rd</sup>, 2<sup>nd</sup>, 1<sup>st</sup> ).
6. Program to find greatest number among three numbers
7. Program which accept Sale Price, Cost Price from user and tell shopkeeper is in Profit, in Loss or neither in Profit nor in Loss.
8. Program to print table of any number
9. Program to print factorial of any number
10. Program to print following series : 1,4,9,16,25,36,49
11. Program to print reverse counting from 100 to 1
12. Program to check a string is palindrome or not
13. Program to find greatest number among 10 numbers
14. Program to print Fibonacci series upto 10 terms (1,1,2,3,5,8,13,21,34,55)
15. Program to check a given character is vowel or not
16. Program to find sum of even and odd numbers separately from given 10 numbers
17. Program to count and find the sum of negative & positive numbers separately.
18. Program to find greatest number in a given list of integers.

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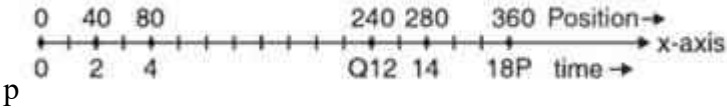
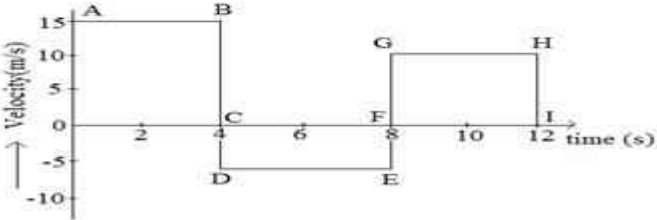
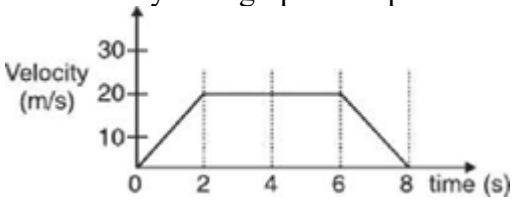
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1	What is the difference between A° and A.U.?
2	Vander Waal's equation $\left(\frac{P+a}{V^2}\right)(V-b) = RT$ find dimensions of a and b
3	E, m, l and G denote energy, mass, angular momentum and gravitational constant respectively. Determine the dimensions of $EL^2 / m^5G^2$
4	Velocity of a body at time t is given by $v = (3t - 2)$ m/s Calculate its i) Acceleration at t = 2s, ii) Average acceleration between t = 0s to t = 2s, iii) Displacement of the body up to 2s.
5	The displacement x of a particle varies with time as $x = 4t^2 - 15t + 25$ . Find the position, velocity and acceleration of the particle at t = 0.
6	The V-t graphs of two objects make angle 30° and 60° with the time axis. Find the ratio of their accelerations.
7	A car is moving along x-axis. As shown in figure it moves from 0 to P in 18 seconds and return from P to Q in 6 second. What are the average velocity and average speed of the car in going from (i) O to P (ii) o to P and back to Q 
8	Velocity time graph of a moving particle is shown. Find the displacement (1) 0 – 4 s (2) 0 – 8 (3) 0 12 s from the graph. Also write the differences between distance and displacement. 
9	The velocity time graph for a particle is shown in figure. Draw acceleration time graph from it. 
10	A particle is moving along a straight line and its position is given by the relation $x = (t^3 - 6t^2 - 15t + 40)$ m Find (a) The time at which velocity is zero. (b) Position and displacement of the particle at that point. (c) Acceleration for the particle at the line

11	A body is thrown with a velocity of 20m/s at an angle $30^{\circ}$ with horizontal. How high will it go? Where and when will it touch the ground?
12	Find the equation of motion of a oblique/angular projectile. Hence show that path of a projectile is parabolic. Also calculate the range, maximum height, time of flight. b) Find the range, time of flight, velocity at any point in case of horizontal projectile.
13	The equation of motion of a body moving in a straight line is given by $S = 2t + 5t^2$ where S is displacement at time t. What is the velocity and acceleration of a body at time $t = 5s$ . Is it a uniform velocity motion or uniform acceleration motion?
14	The magnitude of resultant vector is equal to either of them. What will be the angle between two vectors ?
15	Find the angle between two vectors $2\hat{i} - 2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + 3\hat{k}$ ? Also find a vector perpendicular to both of these.
16	The position (x) of a body at time (t) in SI unit is given by the relation $t = \sqrt{x} - 5$ . Calculate its velocity at time $t = 2s$ .
17	What is the angle of projection at which horizontal range and maximum height are equal?
18	Two forces 5 and 10 kg wt are acting with an inclination of $120^{\circ}$ between them. What is the angle which the resultant makes with 10kg wt?
19	A car moves in such a way that its position changes with time as per the relation $X = 3t^2 - 4t + 1$ , where x is in meter and t is in second. Find average velocity of the car between $t = 1s$ to $t = 3s$ . Also find its velocity at time $t = 2s$ . Hints: $v = dx/dt = 8m/s$
20	Derive expressions for velocity and acceleration for uniform circular motion. Derive expression for linear acceleration in uniform circular motion
21	Derive an equation for the path of a projectile fired parallel to horizontal.
22	From a certain height above the ground a stone A is dropped gently. Simultaneously another stone B is fired horizontally. Which of the two stones will arrive on the ground earlier?
23	A bomb at rest explodes into three fragments of equal masses. Two fragments fly off at right angles to each other with velocities $9 \text{ ms}^{-1}$ and $12 \text{ ms}^{-1}$ respectively. Calculate the speed of the third fragment.
24	State triangle law of vector addition and using it obtain an expression for the resultant of two vectors acting in same order. Three vectors are having same magnitude and one of them is resultant of the other two. Using the triangle law find the angle between the two vectors other than resultant.
25	What is meant by horizontal range of a projectile? Show that if a projectile is fired at $45^{\circ}$ angle with horizontal it covers maximum horizontal range? What is the maximum range of a projectile fired with speed 30m/s. Also prove that if two projectiles are fired from same point with same speed from ground at $45^{\circ} + \theta$ and $45^{\circ} - \theta$ they have same horizontal range.
26	State Newton's second, law of motion. Express it mathematically and hence obtain a relation between force and acceleration.
27	State Newton's real law of motion. How does it explain law of inertia? Derive law of conservation of momentum from this law. A body of mass 4kg accelerates uniformly from 20m/s to 30m/s in 4 sec. How much force acts on it?

28	<p>a) What is meant by coefficient of friction and angle of friction? Establish the relation between the two?</p> <p>b) State laws of limiting and kinetic friction.</p>
29	State and prove the principle of law of conservation of linear momentum?
30	<p>What is meant by a banked road? Draw the free body diagram of a car taking a turn on a banked road and obtain an expression for its maximum safe speed?</p> <p>On a smooth banked curved track of radius of curvature 10m, a car of mass 500kg can safely turn with a maximum speed of 24m/s. What should be the angle of banking of the track? (<math>\mu=0.2</math>)</p>
31	<p>A man weighing 60 Kg is sitting in a lift. What will be its apparent weight if the Lift</p> <p>i) moves vertically upward with a constant velocity of 20m/s.</p> <p>ii) moves vertically downward with a constant velocity of 20m/s</p> <p>iii) moves vertically upward with a constant acceleration of 2m/s<sup>2</sup></p> <p>iv) moves vertically downward with a constant acceleration of 20m/s<sup>2</sup></p> <p>v) breaks while going up?</p>
32	<p>A man of mass 70 kg stands on a weighing scale in a lift which is moving</p> <p>(a) upwards with a uniform speed of <math>10 \text{ m s}^{-1}</math>,</p> <p>(b) downwards with a uniform acceleration of <math>5 \text{ m s}^{-2}</math>,</p> <p>(c) upwards with a uniform acceleration of <math>5 \text{ m s}^{-2}</math>.</p> <p>What would be the readings on the scale in each case?</p> <p>(d) What would be the reading if the lift mechanism failed and it hurtled down freely under gravity?</p>
33	<p>A monkey of mass 40 kg climbs on a rope (Fig. 5.20) which can stand a maximum tension of 600 N. In which of the following cases will the rope break: the monkey</p> <p>(a) climbs up with an acceleration of <math>6 \text{ m s}^{-2}</math></p> <p>(b) climbs down with an acceleration of <math>4 \text{ m s}^{-2}</math></p> <p>(c) climbs up with a uniform speed of <math>5 \text{ m s}^{-1}</math></p> <p>(d) falls down the rope nearly freely under gravity?</p>
34	<p>Define impulse of a force. Write the three applications with explanations.</p> <p>State the law of conservation of linear momentum. Write one application of it.</p>
35	A cricket ball of mass 150g is moving with a velocity of 12m/s and is hit by a bat so that the ball is turned back with a velocity of 20m/s. The force of the blow acts on the ball for 0.01 s. Find the average force exerted on the ball by the bat.
36	What is impulse? Give its SI unit. Show that impulse is equal to change in momentum. Two billiard balls each of mass 0.05kg moving in opposite directions with speed 6m/s collide and rebound with the same speed. What is the impulse imparted to each ball due to other ?
37	A shell of mass 0.020kg is fired by a gun of mass 100kg. If the muzzle speed of the shell is 80m/s, what is the recoil speed of the gun?
38	Force of 16N and 12N are acting on a mass of 200kg in mutually perpendicular directions. Find the magnitude of the acceleration produced?
39	A monkey of mass 40 kg climbs on a rope which can stand a maximum tension 600 N. In which of the following cases will the rope break? The monkey (a) climbs up with an acceleration of 6m/s <sup>2</sup> (b) climbs down with an acceleration of 4m/s <sup>2</sup> (c) climbs up with a uniform speed of 5m/s (d) falls down the rope freely under gravity. Take $g = 10\text{m/s}^2$ and ignore the mass of the rope.
40	A nucleus is at rest in the laboratory frame of reference. Show that if it disintegrates into two smaller nuclei the products must move in opposite directions.







**\*\* Power Set :** The collection of all subsets of a set A is called the power set of A. It is denoted by P(A)

If A is a set with  $n(A) = m$ , then it can be shown that  $n [ P(A) ] = 2^m$ .

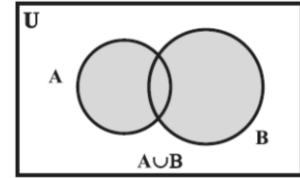
**\*\* Universal Set :** The largest set under consideration is called Universal set.

**\*\* Union of sets :** The union of two sets A and B is the set C which consists of all those elements which are either in A or in B (including those which are in both). In symbols, we write.

$$A \cup B = \{ x : x \in A \text{ or } x \in B \}.$$

$$x \in A \cup B \Rightarrow x \in A \text{ or } x \in B$$

$$x \notin A \cup B \Rightarrow x \notin A \text{ and } x \notin B$$



**\*\* Some Properties of the Operation of Union**

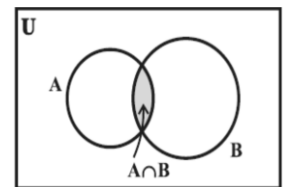
- (i)  $A \cup B = B \cup A$  (Commutative law)
- (ii)  $(A \cup B) \cup C = A \cup (B \cup C)$  (Associative law )
- (iii)  $A \cup \phi = A$  (Law of identity element,  $\phi$  is the identity of  $\cup$ )
- (iv)  $A \cup A = A$  (Idempotent law)
- (v)  $U \cup A = U$  (Law of U)

**\*\* Intersection of sets :** The intersection of two sets A and B is the set of all those elements which belong to both A and B.

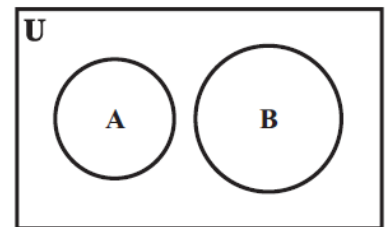
Symbolically, we write  $A \cap B = \{x : x \in A \text{ and } x \in B\}$

$$x \in A \cap B \Rightarrow x \in A \text{ and } x \in B$$

$$x \notin A \cap B \Rightarrow x \notin A \text{ or } x \notin B$$



**\*\* Disjoint sets :** If A and B are two sets such that  $A \cap B = \phi$ , then A and B are called disjoint sets.



**\*\* Some Properties of Operation of Intersection**

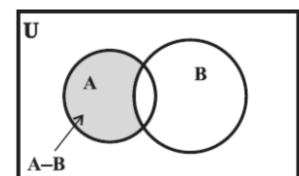
- (i)  $A \cap B = B \cap A$  (Commutative law).
- (ii)  $(A \cap B) \cap C = A \cap (B \cap C)$  (Associative law).
- (iii)  $\phi \cap A = \phi, U \cap A = A$  (Law of  $\phi$  and U).
- (iv)  $A \cap A = A$  (Idempotent law)
- (v)  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$  (Distributive law) i. e.,  $\cap$  distributes

over  $\cup$

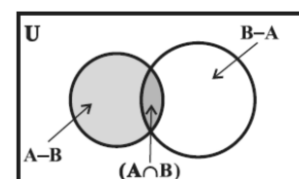
**\*\* Difference of sets :** The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

Symbolically, we write  $A - B$  and read as “ A minus B”.

$$A - B = \{ x : x \in A \text{ and } x \notin B \}.$$

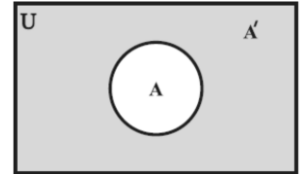


\* The sets  $A - B, A \cap B$  and  $B - A$  are mutually disjoint sets, i.e., the intersection of any of these two sets is the null set.



**\*\* Complement of a Set :** Let  $U$  be the universal set and  $A$  a subset of  $U$ . Then the complement of  $A$  is the set of all elements of  $U$  which are not the elements of  $A$ . Symbolically, we write  $A'$  to denote the complement of  $A$  with respect to  $U$ .

Thus,  $A' = \{x : x \in U \text{ and } x \notin A\}$ . Obviously  $A' = U - A$



**\*\* Some Properties of Complement Sets**

1. Complement laws: (i)  $A \cup A' = U$  (ii)  $A \cap A' = \phi$
2. De Morgan's law: (i)  $(A \cup B)' = A' \cap B'$  (ii)  $(A \cap B)' = A' \cup B'$
3. Law of double complementation :  $(A')' = A$
4. Laws of empty set and universal set  $\phi' = U$  and  $U' = \phi$ .

**\*\* Practical Problems on Union and Intersection of Two Sets :**

- (i)  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- (ii)  $n(A \cup B) = n(A) + n(B)$ , if  $A \cap B = \phi$ .
- (iii)  $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$ .

## QUESTIONS FOR HHW

**1 : How many elements are there in the complement of set A?**

- A. 0                      B. 1                      C. All the elements of A                      D. None of these

**2 : Let  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ,  $P = \{1, 2, 5\}$ ,  $Q = \{6, 7\}$ . Then  $P \cap Q'$  is :**

- A. P                      B. Q                      C.  $Q'$                       D. None

**3. The cardinality of the power set of  $\{x: x \in \mathbb{N}, x \leq 10\}$  is \_\_\_\_\_.**

- A. 1024                      B. 1023                      C. 2048                      D. 2043

**4. If A, B and C are any three sets, then  $A \times (B \cup C)$  is equal to:**

- A.  $(A \times B) \cup (A \times C)$                       C.  $(A \times B) \cap (A \times C)$   
 B.  $(A \cup B) \times (A \cup C)$                       D. None of the above

## (B) ASSERTION AND REASON

1. **DIRECTION:** In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as

- (a) Both assertion and reason are true and reason is the correct explanation of assertion.
- (b) Both assertion and reason are true but reason is not the correct explanation of assertion.

(c) Assertion is true but reason is false.

(d) Assertion is false but reason is true

1. Assertion (A) 'The collection of all natural numbers less than 100' is a set.

Reason (R) :A set is a well-defined collection of the distinct objects.

## Day 2 (21/10/23)

### RELATIONS & FUNCTIONS CONCEPTS AND RESULTS

\*\* **Cartesian Products of Sets** : Given two non-empty sets P and Q. The cartesian product  $P \times Q$  is the set of all ordered pairs of elements from P and Q, i.e.,  $P \times Q = \{ (p, q) : p \in P, q \in Q \}$

\*\* Two ordered pairs are equal, if and only if the corresponding first elements, are equal and the second elements are also equal.

\*\* If there are p elements in A and q elements in B, then there will be pq elements in  $A \times B$ , i.e.

if  $n(A) = p$  and  $n(B) = q$ , then  $n(A \times B) = pq$ .

\*\* If A and B are non-empty sets and either A or B is an infinite set, then so is  $A \times B$ .

\*\*  $A \times A \times A = \{(a, b, c) : a, b, c \in A\}$ . Here (a, b, c) is called an ordered triplet.

\*\* **Relation** : A relation R from a non-empty set A to a non-empty set B is a subset of the cartesian product

$A \times B$ . The subset is derived by describing a relationship between the first element and the second

element of the ordered pairs in  $A \times B$ . The second element is called the image of the first element.

\*\* The set of all first elements of the ordered pairs in a relation R from a set A to a set B is called the

domain of the relation R.

\*\* The set of all second elements in a relation R from a set A to a set B is called the range of the relation

R. The whole set B is called the co-domain of the relation R.  $\text{Range} \subseteq \text{co-domain}$ .

\*\* A relation may be represented algebraically either by the Roster method or by the Set-builder method.

\*\* An arrow diagram is a visual representation of a relation.

\*\* The total number of relations that can be defined from a set A to a set B is the number of possible

subsets of  $A \times B$ . If  $n(A) = p$  and  $n(B) = q$ , then  $n(A \times B) = pq$  and the total number of relations is  $2^{pq}$ .

\*\* A relation R from A to A is also stated as a relation on A.

\*\* **Function**: A relation f from a set A to a set B is said to be a function if every element of set A has one and only one image in set B.

In other words, a function  $f$  is a relation from a non-empty set  $A$  to a non-empty set  $B$  such that

the domain of  $f$  is  $A$  and no two distinct ordered pairs in  $f$  have the same first element.

If  $f$  is a function from  $A$  to  $B$  and  $(a, b) \in f$ , then  $f(a) = b$ , where  $b$  is called the image of  $a$  under

$f$  and  $a$  is called the pre-image of  $b$  under  $f$ .

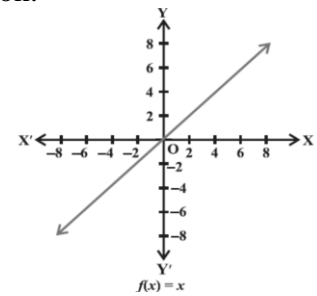
\*\* A function which has either  $\mathbf{R}$  or one of its subsets as its range is called a real valued function. Further,

if its domain is also either  $\mathbf{R}$  or a subset of  $\mathbf{R}$ , it is called a real function.

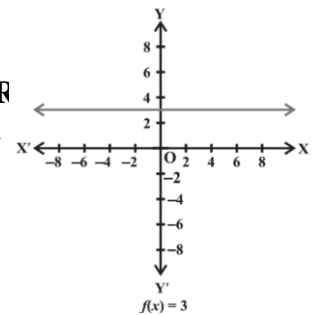
### Some functions and their graphs

\*\* **Identity function** Let  $\mathbf{R}$  be the set of real numbers. Define the real valued function  $f : \mathbf{R} \rightarrow \mathbf{R}$  by  $y = f(x) = x$  for each  $x \in \mathbf{R}$ .

Such a function is called the identity function. Here the domain and range of  $f$  are  $\mathbf{R}$ .



\*\* **Constant function** : Define the function  $f : \mathbf{R} \rightarrow \mathbf{R}$  by  $y = f(x) = c$ ,  $x \in \mathbf{R}$  where  $c$  is a constant and each  $x \in \mathbf{R}$ . Here domain of  $f$  is  $\mathbf{R}$  and its range is  $\{c\}$ .



\*\* **Polynomial function** : A function  $f : \mathbf{R} \rightarrow \mathbf{R}$  is said to be polynomial function if for each  $x$  in  $\mathbf{R}$ ,

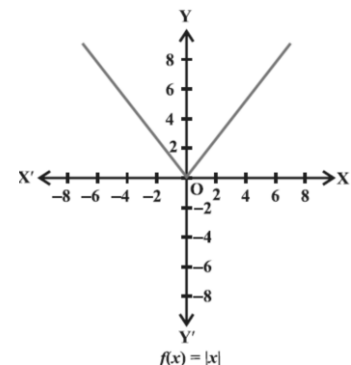
$y = f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ , where  $n$  is a non-negative integer and  $a_0, a_1, a_2, \dots, a_n \in \mathbf{R}$ .

\*\* **Rational functions** : are functions of the type  $\frac{f(x)}{g(x)}$ , where  $f(x)$  and  $g(x)$  are polynomial functions of  $x$  defined in a domain, where  $g(x) \neq 0$ .

\*\* **The Modulus function** : The function  $f : \mathbf{R} \rightarrow \mathbf{R}$  defined by  $f(x) = |x|$  for each  $x \in \mathbf{R}$  is called modulus function. For each non-negative value of  $x$ ,  $f(x)$  is equal to  $x$ .

But for negative values of  $x$ , the value of  $f(x)$  is the negative of

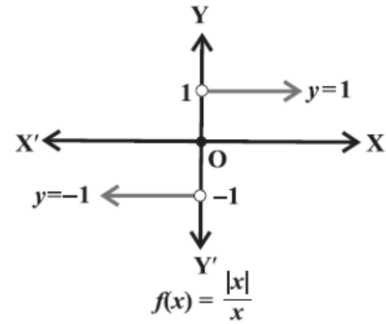
the value of  $x$ , i.e.,  $f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$ .



**\*\* Signum function :** The function  $f : \mathbf{R} \rightarrow \mathbf{R}$  defined by

$$f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$$

is called the signum function. The domain of the signum function is  $\mathbf{R}$  and the range is the set  $\{-1, 0, 1\}$ .

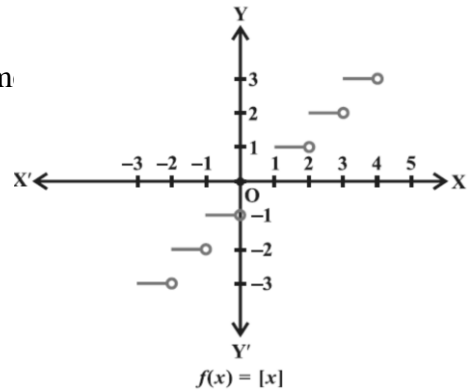


**\*\* Greatest integer function :**

The function  $f : \mathbf{R} \rightarrow \mathbf{R}$  defined by  $f(x) = [x]$ ,  $x \in \mathbf{R}$  assumes the value of the greatest integer, less than or equal to  $x$ .

Such a function is called the greatest integer function.

$$\begin{aligned} [x] &= -1 \text{ for } -1 \leq x < 0 \\ [x] &= 0 \text{ for } 0 \leq x < 1 \\ [x] &= 1 \text{ for } 1 \leq x < 2 \\ [x] &= 2 \text{ for } 2 \leq x < 3 \text{ and so on.} \end{aligned}$$



### Algebra of real functions

**\*\* Addition of two real functions :** Let  $f : X \rightarrow \mathbf{R}$  and  $g : X \rightarrow \mathbf{R}$  be any two real functions, where  $X \subset \mathbf{R}$ .

Then, we define  $(f + g) : X \rightarrow \mathbf{R}$  by  $(f + g)(x) = f(x) + g(x)$ , for all  $x \in X$ .

**\*\* Subtraction of a real function from another :** Let  $f : X \rightarrow \mathbf{R}$  and  $g : X \rightarrow \mathbf{R}$  be any two real functions,

where  $X \subset \mathbf{R}$ . Then, we define  $(f - g) : X \rightarrow \mathbf{R}$  by  $(f - g)(x) = f(x) - g(x)$ , for all  $x \in X$ .

**\*\* Multiplication by a scalar :** Let  $f : X \rightarrow \mathbf{R}$  be a real valued function and  $\alpha$  be a scalar. Here by scalar, we

mean a real number. Then the product  $\alpha f$  is a function from  $X$  to  $\mathbf{R}$  defined by

$$(\alpha f)(x) = \alpha f(x), x \in X.$$

**\*\* Multiplication of two real functions :** The product (or multiplication) of two real functions  $f : X \rightarrow \mathbf{R}$  and

$g : X \rightarrow \mathbf{R}$  is a function  $fg : X \rightarrow \mathbf{R}$  defined by  $(fg)(x) = f(x)g(x)$ , for all  $x \in X$ .

**\*\* Quotient of two real functions** Let  $f$  and  $g$  be two real functions defined from  $X \rightarrow \mathbf{R}$  where  $X \subset \mathbf{R}$ . The

$$\text{quotient of } f \text{ by } g \text{ denoted by } \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \text{ provided } g(x) \neq 0, x \in X$$

### QUESTIONS FOR HHW

- Q 1. If  $A = \{a, b\}$  and  $B = \{1, 2\}$  then the number of functions from set  $A$  to set  $B$  is  
 (A) 2 (B) 4 (C) 16 (D) None

of these

- Q 2. A function is defined by  $f(t) = 2t - 5$ , then the value of  $f(-3)$  is

- (A) - 11                      (B) 11                      (C) 1                      (D) -1
- Q 3. If  $f(x) = -|x|$ . Choose the correct option from the following:  
 (A) Domain is set of negative real numbers                      (B) Range is set of real numbers  
 (C) Range is set of all negative integers                      (D) Range is  $(-\infty, 0]$
- Q 4. Let  $f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$  be a function from  $Z$  to  $Z$  defined by  $f(x) = mx + c$ . Determine  $c$ .  
 (A) 1                      (B) 0                      (C) - 1                      (D) - 3

**Assertion and Reason type (1 mark)**

- Q 5. In the following question, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.  
 (a) Both (A) and (R) are true and (R) is the correct explanation of (A).  
 (b) Both (A) and (R) are true but (R) is not the correct explanation of (A).  
 (c) (A) is true but (R) is false.  
 (d) (A) is false but (R) is true.

ASSERTION (A): The function  $f: A \rightarrow B$  defined by  $f = \{(1, x), (2, y), (3, x)\}$ , then its domain is  $A = \{1, 2, 3\}$  and range is  $\{x, y\}$ .

REASON (R): The range of the function  $f$  is always the co-domain set.

## Day 3(22/10/23)

### TRIGONOMETRIC FUNCTIONS CONCEPTS AND RESULTS

**Angles :** Angle is a measure of rotation of a given ray about its initial point.

**\*\* Measurement of an angle.**

**\*\*English System (Sexagesimal system)**

(i) 1 right angle = 90 degrees =  $90^\circ$ .    (ii)  $1^\circ = 60$  minutes =  $60'$ .    (iii)  $1' = 60$  second =  $60''$ .

**\*\*French System (Centesimal system)**

(iv) 1 right angle = 100 grades = 100 g.    (v) 1 g = 100 minutes = 100 ‘    (vi)  $1' = 100$  seconds = 100 ‘‘

**\*\*Circular System.**

(vii)  $180^\circ = 200^g = \pi$  radians = 2 right angles, where a radian is an angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle.

(viii) The circular measure  $\theta$  of an angle subtended at the centre of a circle by an arc of length  $l$  is equal to the ratio of the length  $l$  to the radius  $r$  of the circle.

(ix) Each interior angle of a regular polygon of  $n$  sides is equal to  $\frac{2n-4}{n}$  right angles.

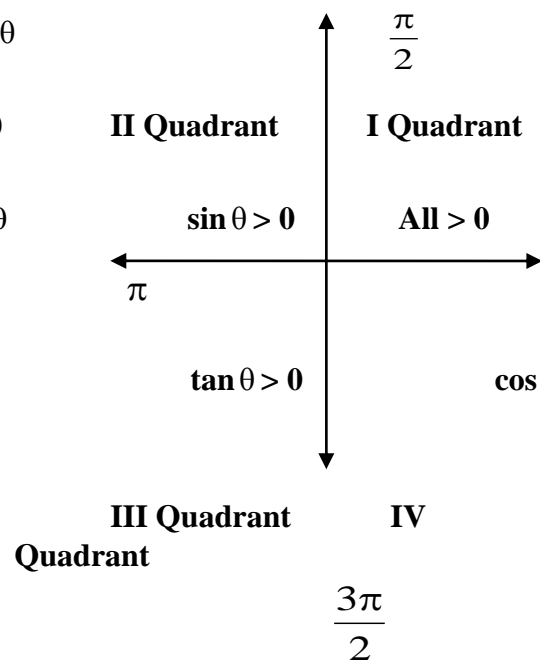
T-ratios	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
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Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	n.d	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

**\*\* Formulae for t-ratios of Allied Angles :**

All T-ratio changes in  $\frac{\pi}{2} \pm \theta$  and  $\frac{3\pi}{2} \pm \theta$  while remains unchanged in  $\pi \pm \theta$  and  $2\pi \pm \theta$ .

$$\begin{aligned} \sin\left(\frac{\pi}{2} \pm \theta\right) &= \cos \theta & \sin\left(\frac{3\pi}{2} \pm \theta\right) &= -\cos \theta \\ \cos\left(\frac{\pi}{2} \pm \theta\right) &= \mp \sin \theta & \cos\left(\frac{3\pi}{2} \pm \theta\right) &= \pm \sin \theta \\ \tan\left(\frac{\pi}{2} \pm \theta\right) &= \mp \cot \theta & \tan\left(\frac{3\pi}{2} \pm \theta\right) &= \mp \cot \theta \\ \sin(\pi \pm \theta) &= \mp \sin \theta & \sin(2\pi \pm \theta) &= \pm \sin \theta \\ \cos(\pi \pm \theta) &= -\cos \theta & \cos(2\pi \pm \theta) &= \cos \theta \\ \tan(\pi \pm \theta) &= \mp \tan \theta & \tan(2\pi \pm \theta) &= \mp \tan \theta \end{aligned}$$



**\*\* Sum and Difference formulae :**

$$\begin{aligned} \sin(A + B) &= \sin A \cos B + \cos A \sin B \\ \sin(A - B) &= \sin A \cos B - \cos A \sin B \\ \cos(A + B) &= \cos A \cos B - \sin A \sin B \\ \cos(A - B) &= \cos A \cos B + \sin A \sin B \end{aligned}$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}, \quad \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}, \quad \tan\left(\frac{\pi}{4} + A\right) = \frac{1 + \tan A}{1 - \tan A},$$

$$\tan\left(\frac{\pi}{4} - A\right) = \frac{1 - \tan A}{1 + \tan A}, \quad \cot(A + B) = \frac{\cot A \cdot \cot B - 1}{\cot B + \cot A}, \quad \cot(A - B) = \frac{\cot A \cdot \cot B + 1}{\cot B - \cot A}$$

$$\begin{aligned} \sin(A + B) \sin(A - B) &= \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A \\ \cos(A + B) \cos(A - B) &= \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A \end{aligned}$$

**\*\*Formulae for the transformation of a product of two circular functions into algebraic sum of two circular functions and vice-versa.**

$$\begin{aligned} 2 \sin A \cos B &= \sin(A + B) + \sin(A - B) \\ 2 \cos A \sin B &= \sin(A + B) - \sin(A - B) \end{aligned}$$



$$2 \cos A \cos B = \cos(A + B) + \cos(A - B)$$

$$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$

$$\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2},$$

$$\frac{C-D}{2}.$$

$$\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}.$$

$$\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2},$$

$$\frac{C-D}{2}.$$

$$\cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}.$$

**\*\* Formulae for t-ratios of multiple and sub-multiple angles :**

$$\sin 2A = 2 \sin A \cos A = \frac{2 \tan A}{1 + \tan^2 A}.$$

$$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A = 2 \cos^2 A - 1 = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$1 + \cos 2A = 2 \cos^2 A \quad 1 - \cos 2A = 2 \sin^2 A \quad 1 + \cos A = 2 \cos^2 \frac{A}{2} \quad 1 - \cos A = 2 \sin^2 \frac{A}{2}$$

$$\sin^2 \frac{A}{2}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A},$$

$$\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}.$$

$$\sin 3A = 3 \sin A - 4 \sin^3 A,$$

$$\cos 3A = 4 \cos^3 A - 3 \cos A$$

$$\sin 15^\circ = \cos 75^\circ = \frac{\sqrt{3} - 1}{2\sqrt{2}}.$$

$$\& \quad \cos 15^\circ = \sin 75^\circ = \frac{\sqrt{3} + 1}{2\sqrt{2}},$$

$$\tan 15^\circ = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = 2 - \sqrt{3} = \cot 75^\circ$$

$$\& \quad \tan 75^\circ = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} = 2 + \sqrt{3} = \cot 15^\circ.$$

$$\sin 18^\circ = \frac{\sqrt{5} - 1}{4} = \cos 72^\circ$$

$$\text{and } \cos 36^\circ = \frac{\sqrt{5} + 1}{4} = \sin 54^\circ.$$

$$\sin 36^\circ = \frac{\sqrt{10 - 2\sqrt{5}}}{4} = \cos 54^\circ$$

$$\text{and } \cos 18^\circ = \frac{\sqrt{10 + 2\sqrt{5}}}{4} = \sin 72^\circ.$$

$$\tan \left( 22 \frac{1}{2} \right)^\circ = \sqrt{2} - 1 = \cot 67 \frac{1}{2}^\circ$$

$$\text{and } \tan \left( 67 \frac{1}{2} \right)^\circ = \sqrt{2} + 1 = \cot \left( 22 \frac{1}{2} \right)^\circ.$$

**\*\* Properties of Triangles :** In any  $\Delta ABC$ ,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad [\text{Sine Formula}]$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}, \quad \cos B = \frac{c^2 + a^2 - b^2}{2ca}, \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}.$$

**\*\* Projection Formulae :**  $a = b \cos C + c \cos B$ ,  $b = c \cos A + a \cos C$ ,  $c = a \cos B + b \cos A$

**\*\* Some important trigonometric substitutions :**

$\sqrt{a^2 + x^2}$	Put $x = a \tan \theta$ or $a \cot \theta$
$\sqrt{x^2 - a^2}$	Put $x = a \sec \theta$ or $a \operatorname{cosec} \theta$
$\sqrt{a+x}$ or $\sqrt{a-x}$ or both	Put $x = a \cos 2\theta$
$\sqrt{a^n + x^n}$ or $\sqrt{a^n - x^n}$ or both	Put $x^n = a^n \cos 2\theta$
$\sqrt{1 + \sin 2\theta}$	$= \sin \theta + \cos \theta$
$\sqrt{1 - \sin 2\theta}$	$= \cos \theta - \sin \theta, 0 < \theta < \frac{\pi}{4}$
	$= \sin \theta - \cos \theta, \frac{\pi}{4} < \theta < \frac{\pi}{2}$

**\*\*General solutions:**

- \*  $\cos \theta = 0 \Rightarrow \theta = n\pi, n \in \mathbb{Z}$
- \*  $\sin \theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$
- \*  $\tan \theta = 0 \Rightarrow \theta = n\pi, n \in \mathbb{Z}$
- \*  $\sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \alpha, n \in \mathbb{Z}$
- \*  $\cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha, n \in \mathbb{Z}$
- \*  $\tan \theta = \tan \alpha \Rightarrow \theta = n\pi + \alpha, n \in \mathbb{Z}$

**Examples:**

**1. The value of  $\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ$  is:**

- (a) 0            (b) 1            (c)  $\frac{1}{2}$             (d) Not defined

Correct option: (b) 1

**Solution:**  $\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ$

$$= [\tan 1^\circ \tan 2^\circ \dots \tan 44^\circ] \tan 45^\circ [\tan (90^\circ - 44^\circ) \tan (90^\circ - 43^\circ) \dots \tan (90^\circ - 1^\circ)]$$

$$= [\tan 1^\circ \tan 2^\circ \dots \tan 44^\circ] [\cot 44^\circ \cot 43^\circ \dots \cot 1^\circ] \times [\tan 45^\circ]$$

$$= [(\tan 1^\circ \times \cot 1^\circ) (\tan 2^\circ \times \cot 2^\circ) \dots (\tan 44^\circ \times \cot 44^\circ)] \times [\tan 45^\circ]$$

We know that,  $\tan A \times \cot A = 1$  and  $\tan 45^\circ = 1$

Hence, the equation becomes as;

$$= 1 \times 1 \times 1 \times 1 \times \dots \times 1 = 1 \quad \{\text{As } 1^n = 1\}$$

**2. If  $\alpha + \beta = \pi/4$ , then the value of  $(1 + \tan \alpha)(1 + \tan \beta)$  is :**

- (a) 1            (b) 2            (c) -2            (d) Not defined

Correct option: (b) 2

**Solution:** Given,  $\alpha + \beta = \pi/4$

Taking "tan" on both sides,

$$\tan(\alpha + \beta) = \tan \pi/4$$

We know that,  $\tan(A + B) = (\tan A + \tan B)/(1 - \tan A \tan B)$

and  $\tan \pi/4 = 1$ .

$$\text{So, } (\tan \alpha + \tan \beta)/(1 - \tan \alpha \tan \beta) = 1$$

$$\tan \alpha + \tan \beta = 1 - \tan \alpha \tan \beta$$

$$\tan \alpha + \tan \beta + \tan \alpha \tan \beta = 1 \dots (i)$$

$$(1 + \tan \alpha)(1 + \tan \beta) = 1 + \tan \alpha + \tan \beta + \tan \alpha \tan \beta$$

$$= 1 + 1 \text{ [From (i)]} = 2$$

**3. Find the radius of the circle in which a central angle of  $60^\circ$  intercepts an arc of length**

**37.4 cm (use  $\pi = 22/7$ ).**

**Solution:** Given, Length of the arc =  $l = 37.4$  cm

Central angle =  $\theta = 60^\circ = 60\pi/180$  radian =  $\pi/3$  radians

We know that,  $r = l/\theta$

$$= (37.4) * (\pi / 3) = (37.4) / [22 / 7 * 3] = 35.7 \text{ cm}$$

**Q4. Find the value of  $\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ$ .**

**Solution:**  $\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ$

$$= \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ} = \frac{\sqrt{3} \cos 20^\circ - \sin 20^\circ}{\sin 20^\circ \cos 20^\circ} = 4 \left( \frac{\frac{\sqrt{3}}{2} \cos 20^\circ - \frac{1}{2} \sin 20^\circ}{2 \sin 20^\circ \cos 20^\circ} \right)$$

$$= 4 \left( \frac{\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ}{\sin 40^\circ} \right) = 4 \left( \frac{\sin (60^\circ - 20^\circ)}{\sin 40^\circ} \right) = 4$$

## QUESTIONS FOR HHW

1. A horse is tied to a post by a rope. If the horse moves along a circular path always keeping the rope tight and describes 88 m when it has traced out  $72^\circ$  at the centre, find the length of the rope.
2. If  $\cos \theta = -\frac{1}{2}$ ,  $\pi < \theta < \frac{3\pi}{2}$ , Evaluate  $4 \tan^2 \theta - 3 \operatorname{Cosec}^2 \theta$ .
3. Show that  $\cos 60^\circ + \cos 120^\circ + \cos 240^\circ - \sin 330^\circ = 0$
4. Show that  $\sqrt{2 + \sqrt{2 + 2 \cos 4x}} = 2 \cos x$
5. Show that  $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4 \sin^2 \left( \frac{x-y}{2} \right)$
6. Show that  $\cos 2\theta \cdot \cos \frac{\theta}{2} - \cos 3\theta \cdot \cos \frac{9\theta}{2} = \sin 5\theta \cdot \sin \frac{5\theta}{2}$
7. Show that  $\frac{1 + \sin 2x - \cos 2x}{1 + \sin 2x + \cos 2x} = \tan x$
8. Show that  $\cos A \cdot \cos (60 - A) \cdot \cos (60 + A) = \frac{\cos 3A}{4}$
9. If  $\cos A + \cos B = \frac{1}{2}$ ,  $\sin A + \sin B = \frac{1}{4}$ , Show that  $\tan \left( \frac{A+B}{2} \right) = \frac{1}{2}$
10. Show that  $2 \sin^2 \beta + 4 \cos(\alpha + \beta) \sin \alpha \sin \beta + \cos 2(\alpha + \beta) = \cos 2\alpha$ .

# DAY 4(23/10/23)

## COMPLEX NUMBERS CONCEPTS AND RESULTS

\* A number of the form  $(a + ib)$  where  $a, b \in \mathbb{R}$ , the set of real numbers, and  $i = \sqrt{-1}$  (iota) is called a

complex number. It is denoted by  $z$ ,  $z = a + ib$ . “ $a$ ” is called the real part of complex number  $z$  and “ $b$ ”

is the imaginary part i.e.  $\text{Re}(z) = a$  and  $\text{Im}(z) = b$ .

\* Two complex numbers are said to be equal i.e.  $z_1 = z_2$ .

$$\Leftrightarrow (a + ib) = (c + id)$$

$$\Leftrightarrow a = c \text{ and } b = d$$

$$\Leftrightarrow \text{Re}(z_1) = \text{Re}(z_2) \ \& \ \text{Im}(z_1) = \text{Im}(z_2).$$

\* A complex number  $z$  is said to be purely real if  $\text{Im}(z) = 0$

and is said to be purely imaginary if  $\text{Re}(z) = 0$ .

\* The set  $\mathbb{R}$  of real numbers is a proper subset of the set of complex number  $\mathbb{C}$ , because every real number

can be considered as a complex number with imaginary part zero.

$$* i^{4n} = (i^4)^n = (1)^n = 1$$

$$i^{4n+1} = i^{4n} \cdot i = (1) \cdot i = i$$

$$i^{4n+2} = i^{4n} \cdot i^2 = (1)(-1) = -1$$

$$i^{4n+3} = i^{4n} \cdot i^3 = (1)(-i) = -i.$$

### Algebra of Complex Numbers

\*\* **Addition of two complex numbers** : Let  $z_1 = a + ib$  and  $z_2 = c + id$  be any two complex numbers.

Then, the sum  $z_1 + z_2$  is defined as follows:  $z_1 + z_2 = (a + c) + i(b + d)$ , which is again a complex number.

The addition of complex numbers satisfy the following properties:

(i) The closure law The sum of two complex numbers is a complex number, i.e.,  $z_1 + z_2$  is a complex number for all complex numbers  $z_1$  and  $z_2$ .

(ii) The commutative law For any two complex numbers  $z_1$  and  $z_2$ ,  $z_1 + z_2 = z_2 + z_1$

(iii) The associative law For any three complex numbers  $z_1, z_2, z_3$ ,  $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$ .

(iv) The existence of additive identity There exists the complex number  $0 + i0$  (denoted as  $0$ ), called the additive identity or the zero complex number, such that, for every complex number  $z$ ,  $z + 0 = z$ .

(v) The existence of additive inverse To every complex number  $z = a + ib$ , we have the complex number  $-a + i(-b)$  (denoted as  $-z$ ), called the additive inverse or negative of  $z$ . Thus  $z + (-z) = 0$  (the additive identity).

\*\* **Difference of two complex numbers** : Given any two complex numbers  $z_1$  and  $z_2$ , the difference  $z_1 - z_2$  is defined as follows:  $z_1 - z_2 = z_1 + (-z_2)$ .

\*\* **Multiplication of two complex numbers** : Let  $z_1 = a + ib$  and  $z_2 = c + id$  be any two complex numbers.

Then, the product  $z_1 z_2$  is defined as follows:  $z_1 z_2 = (ac - bd) + i(ad + bc)$

\*\*The multiplication of complex numbers possesses the following properties :

(i) **The closure law** The product of two complex numbers is a complex number, the product  $z_1 z_2$  is a

complex number for all complex numbers  $z_1$  and  $z_2$ .

(ii) **The commutative law** For any two complex numbers  $z_1$  and  $z_2$ ,  $z_1 z_2 = z_2 z_1$

(iii) **The associative law** For any three complex numbers  $z_1, z_2, z_3$ ,  $(z_1 z_2) z_3 = z_1 (z_2 z_3)$ .

(iv) **The existence of multiplicative identity** There exists the complex number  $1 + i 0$  (denoted as 1), called the multiplicative identity such that  $z \cdot 1 = z$ , for every complex number  $z$ .

(v) **The existence of multiplicative inverse** For every non-zero complex number  $z = a + ib$  or  $a + bi$

( $a \neq 0, b \neq 0$ ), we have the complex number  $\frac{a}{a^2 + b^2} + i \frac{-b}{a^2 + b^2}$  (denoted by  $\frac{1}{z}$  or  $z^{-1}$ ),

called the

multiplicative inverse of  $z$  such that  $z \cdot \frac{1}{z} = 1$  (the multiplicative identity).

(vi) **The distributive law** For any three complex numbers  $z_1, z_2, z_3$ ,

$$(a) z_1 (z_2 + z_3) = z_1 z_2 + z_1 z_3 \quad (b) (z_1 + z_2) z_3 = z_1 z_3 + z_2 z_3$$

**\*\*Division of two complex numbers :** Given any two complex numbers  $z_1$  and  $z_2$ , where  $z_2 \neq 0$ , the quotient  $\frac{z_1}{z_2}$  is defined by  $\frac{z_1}{z_2} = z_1 \cdot \frac{1}{z_2}$ .

**\*\*Modulus a Complex Number :** Let  $z = a + ib$  be a complex number. Then, the modulus of  $z$ , denoted by  $|z|$ , is defined to be the non-negative real number  $\sqrt{a^2 + b^2}$ , i.e.,  $|z| = \sqrt{a^2 + b^2}$

**\*\* Properties of Modulus :**

If  $z, z_1, z_2$  are three complex numbers then

(i)  $|z| = 0 \Leftrightarrow z = 0$  i.e., real part and imaginary part are zeroes.

(ii)  $|z| = |\bar{z}| = |-z|$

(iii)  $z \cdot \bar{z} = |z|^2$

(iv)  $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$

(v)  $\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}, z_2 \neq 0$

(vi)  $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\text{Re}(z_1 \bar{z}_2)$

(vii)  $|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2\text{Re}(z_1 \bar{z}_2)$

(viii)  $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$

**\*\*Conjugate of a Complex Number :** Let  $z = a + ib$  then its conjugate is denoted by  $\bar{z} = (a - ib)$ .

**\*\*Properties of conjugates :**

(i)  $\overline{(\bar{z})} = z$

(ii)  $z + \bar{z} = 2\text{Re}(z)$

(iii)  $z - \bar{z} = 2i\text{Im}(z)$

(iv)  $z + \bar{z} = 0 \Rightarrow z$  is purely real.

(v)  $z \cdot \bar{z} = [\text{Re}(z)]^2 + [\text{Im}(z)]^2.$

(vi)  $\overline{z_1 \pm z_2} = \overline{z_1} \pm \overline{z_2}$

(vii)  $\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$

(viii)  $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}, \quad z_2 \neq 0$

**\*\*Argand Plane and Polar Representation**

Some complex numbers such as  $2 + 4i, -2 + 3i, 0 + 1i, 2 + 0i, -5 - 2i$  and  $1 - 2i$  which correspond to the ordered pairs  $(2, 4), (-2, 3), (0, 1), (2, 0), (-5, -2),$  and  $(1, -2),$  respectively, have been represented geometrically by the points A, B, C, D, E, and F, respectively.

The plane having a complex number assigned to each of its point is called the complex plane or the Argand plane.

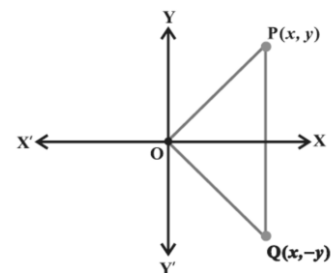
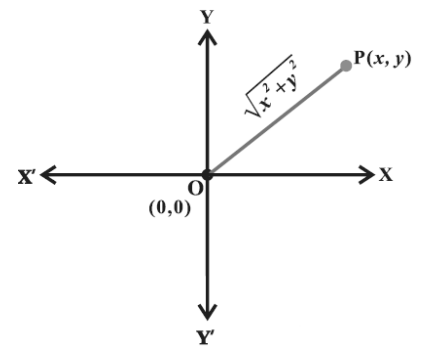
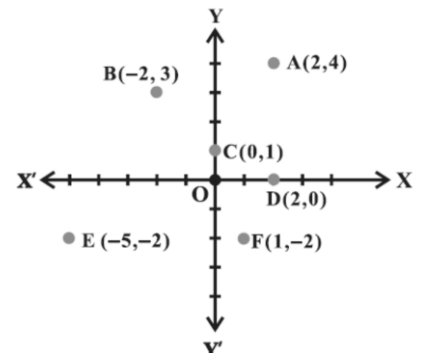
In the Argand plane, the modulus of the complex number

$x + iy = \sqrt{x^2 + y^2}$  is the distance between the point  $P(x, y)$  to the origin  $O(0, 0)$ . The points on the x-axis corresponds to the complex numbers of the form  $a + i0$  and the points on the y-axis corresponds to the complex numbers of the form  $0 + i b$ .

The x-axis and y-axis in the Argand plane are called, respectively, the real axis and the imaginary axis.

The representation of a complex number  $z = x + iy$  and its conjugate  $\bar{z} = x - iy$  in the Argand plane are, respectively, the points  $P(x, y)$  and  $Q(x, -y)$ .

Geometrically, the point  $(x, -y)$  is the mirror image of the point  $(x, y)$  on the real axis



**QUESTIONS FOR HHW**

1	$i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is equal to (a)0 (b) 1 (c) -1 (d) 2
2	If $z_1 = 3 + 2i$ and $z_2 = 2 - 4i$ and $ z_1 + z_2 ^2 +  z_1 - z_2 ^2$ is equal (a) 11 (b)22 (c) 55 (d)66
3	The real part of $\frac{(1+i)^2}{3-i}$ is (a) $\frac{1}{3}$ (b) $-\frac{1}{5}$ (c) $-\frac{1}{3}$ (d) None of these
4	If $z = -5i^{-15} - 6i^{-8}$ then $\bar{z}$ is equal to (a) -6-5i (b) -6 +5i (c) 6 -5i (d) 6 +5i
5	Multiplicative Inverse of complex number $(1-2i)=...$ (a) $\frac{1}{5} + i\frac{2}{5}$ (b) $\frac{1}{5} - i\frac{2}{5}$ (c) $-\frac{1}{5} + i\frac{2}{5}$ (d) None of these
6	<b>Assertion-Reason</b> Assertion: The equation $ix^2 - 3ix + 2i = 0$ has non real roots.

	<p>Reason: If <math>a, b, c</math> are real and <math>b^2 - 4ac \geq 0</math>, then the roots of the equation <math>ax^2 + bx + c = 0</math> are real and if <math>b^2 - 4ac &lt; 0</math>, then the roots of the equation <math>ax^2 + bx + c = 0</math> are non-real.</p> <p>(a) A is true, R is true; R is a correct explanation of A.  (b) A is true, R is true; R is not a correct explanation of A.  (c) A is true; R is false  (d) A is false; R is true.</p>
7	<p><b>Case study</b></p> <p>The conjugate of a complex number <math>z</math> is the complex number obtained by replacing <math>i</math> with <math>-i</math> number. It is denoted by <math>\bar{z}</math>.  The modulus of a complex number <math>z = a + ib</math> is defined as the non-negative real number <math>\sqrt{a^2 + b^2}</math>. It is denoted by <math> z </math> i. e  <math> z  = \sqrt{a^2 + b^2}</math></p> <p>(a) If <math>(x - iy)(3 + 5i)</math> is the conjugate of <math>-6 - 24i</math>, then find the value of <math>x + y</math>  (b) If <math>f(z) = \frac{7-z}{1-z^2}</math>, where <math>z = 1 + 2i</math> then find <math> f(z) </math>.</p>
8	<p>Express the following complex number in the form <math>a + ib</math></p> $\frac{3 - \sqrt{-16}}{1 - \sqrt{-9}}$
9	Evaluate $1 + i^2 + i^4 + i^6 + \dots + i^{20}$ .
10	If $z_1, z_2$ are $1 - i$ and $-2 + 4i$ respectively find $\text{Im}\left(\frac{z_1 z_2}{z_1}\right)$
11	Find the value of $(1 + i)^6 + (1 - i)^3$
12	Solve the equation $ z + 1  = z + 2(1 + i)$
13	If $z = x + iy$ and $w = \frac{1-iz}{ z-i }$ and $ w  = 1$ then show that $z$ is purely real.
14	If $x+iy = \sqrt{\frac{a+ib}{c+id}}$ , then prove that $(x^2 + y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$

## DAY 5(24/10/23)

### LINEAR INEQUALITIES MAIN CONCEPTS AND RESULTS

\* Two real numbers or two algebraic expressions related by the symbol ' $<$ ', ' $>$ ', ' $\leq$ ' or ' $\geq$ ' form an

**inequality.**

\* **Numerical inequalities :**  $3 < 5$ ;  $7 > 5$

\* **Literal inequalities :**  $x < 5$ ;  $y > 2$ ;  $x \geq 3$ ,  $y \leq 4$

\* **Double inequalities :**  $3 < 5 < 7$ ,  $2 < y < 4$

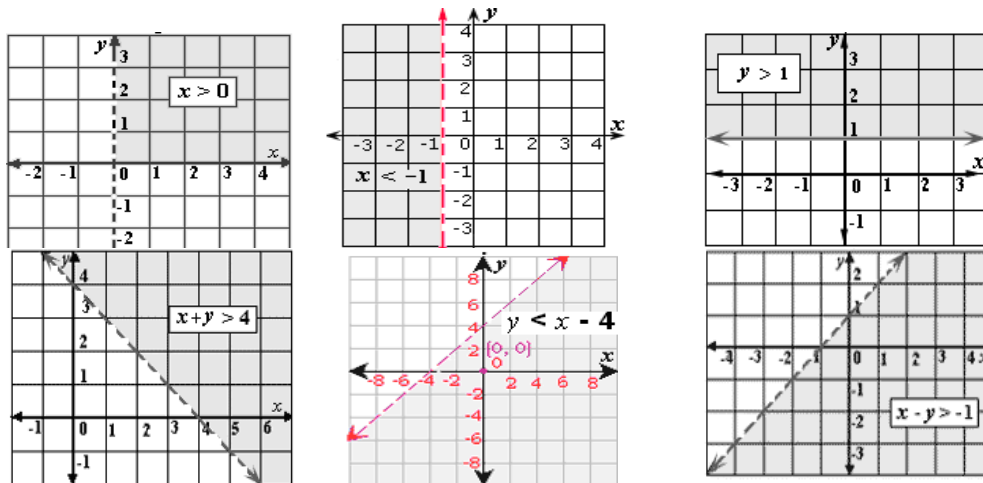
\* **Strict inequalities :**  $ax + b < 0$ ,  $ax + b > 0$ ,  $ax^2 + bx + c > 0$

\* **Slack inequalities :**  $ax + by \leq c$  ,  $ax + by \geq c$  ,  $ax^2 + bx + c \leq 0$

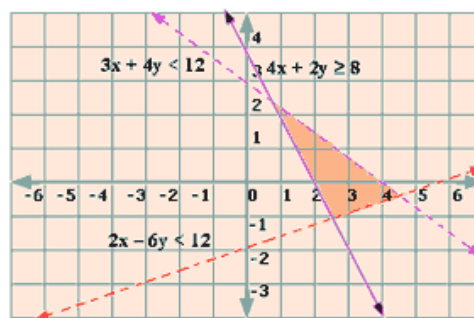
\* **Linear inequalities :**  $ax + b < 0$  ,  $ax + b \geq 0$

\* **Quadratic inequalities :**  $ax^2 + bx + c > 0$  ,  $ax^2 + bx + c \leq 0$

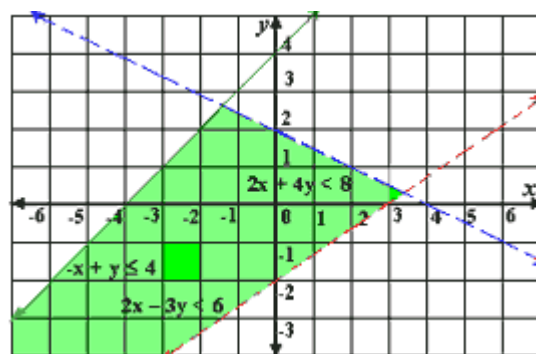
**\*\* Algebraic Solutions of Linear Inequalities in One Variable and their Graphical Representation**



**\*\* Graph of system of linear inequalities,  $2x - 6y < 12$ ,  $3x + 4y < 12$  and  $4x + 2y \geq 8$ .**



**\*\*Graph the system of linear inequalities.  $2x - 3y < 6$  ,  $-x + y \leq 4$  ,  $2x + 4y < 8$**



**QUESTIONS FOR HHW**

1. Given that  $x, y$  and  $b$  are real numbers and  $x < y$ ,  $b > 0$  , then

- A.  $\frac{x}{b} < \frac{y}{b}$       B.  $\frac{x}{b} \leq \frac{y}{b}$       C.  $\frac{x}{b} > \frac{y}{b}$       D.  $\frac{x}{b} \geq \frac{y}{b}$

2. The solution set of equation  $|x + 2| \leq 5$  is



- A.  $(-7,5)$                       B.  $[-7,3]$                       C.  $[-5,5]$                       D.  $(-7,3)$

3. The shaded part of a line is in given figure can also be described as



- A.  $(-\infty, 1) \cup (2, \infty)$     B.  $(-\infty, 1] \cup [2, \infty)$     C.  $(1,2)$                       D.  $[1,2]$

4. A recharger manufacturing company produces rechargers and its cost function for a week is  $C(x) = \frac{1}{10}(4270 + 23x)$  and its revenue function is  $R(x) = 3x$ , where  $x$  is the number of rechargers produced and sold per week. Number of rechargers must be sold for the company to realize a profit is

- A.  $x \geq 618$                       B.  $x > 610$                       C.  $x > 480$                       D. None of These

**In the following questions, a statement of Assertion(A)is followed by a statement of Reason (R). Choose the correct answer out of the following choices.**

- (a) **Both (A) and (R) are true and (R) is the correct explanation of (A).**  
 (b) **Both (A) and (R) are true but (R) is not the correct explanation of (A).**  
 (c) **(A) is true but (R) is false.**  
 (d) **(A) is false but(R) is true.**

5. Assertion (A): The solution set of the inequality  $x-3 < 2, x \in N$  is  $\{1,2,3,4,5,6,7,8\}$ .

Reason (R) :Solution set of a inequality in  $x$  is set of values of  $x$  satisfying the inequality .

Answer . 1.A                      2.B.                      3. A                      4. B                      5. d

Short type questions ( 2marks/3 marks)

1. Solve the inequation  $3x + 17 \leq 2(1 - x)$
2. Solve the inequality  $\frac{x+3}{x-2} \leq 2$
3. Find all pair of consecutive odd integers , both are smaller than 18, such that their sum is more than 20.
4. In a game, a person wins if he gets the sum greater than 20 in four throws of a die. In three throws he got numbers 6, 5, 4. What should be number in his fourth throw, so that he wins the game.
5. Solve the inequalities and represent the solution graphically on number line:  
 $5x + 1 > -24, 5x - 1 < 24$ .
6. Solve  $3x-5 < x+1$ . Show the solution on number line.

7. A solution of 9% acid is to be diluted by adding 3% acid solution to it. The resulting mixture is to be more than 5% but less than 7% acid. If there is 460 liters of 9% acid solution, how many liters of 3% solution will have to be added?

8. Solve the inequality  $\frac{2x-3}{4} + 9 \geq 3 + \frac{4x}{3}$ .

9. The longest side of a triangle is twice the shortest side and the third side is 2 cm longer than the shortest side. If the perimeter of the triangle is more than 166 cm then find the minimum length of the shortest side.

10. Solve the inequation  $\frac{2x+4}{x-1} \geq 5$

## DAY 6(25/10/23)

### PERMUTATIONS AND COMBINATIONS

#### MAIN CONCEPTS AND RESULTS

\*\* Fundamental principle of counting, or( the multiplication principle): “If an event can occur in m different ways, following which another event can occur in n different ways, then the total number of occurrence of the events in the given order is  $m \times n$ .”

\*\***Factorial notation** The notation  $n!$  represents the product of first n natural numbers, i.e., the product

$1 \times 2 \times 3 \times \dots \times (n - 1) \times n$  is denoted as  $n!$ . We read this symbol as ‘n factorial’.

Thus,  $1 \times 2 \times 3 \times 4 \dots \times (n - 1) \times n = n!$

$$n! = n(n - 1)!$$

$$= n(n - 1)(n - 2)! \quad [\text{provided } (n \geq 2)]$$

$$= n(n - 1)(n - 2)(n - 3)! \quad [\text{provided } (n \geq 3)]$$

\*\***Permutations** A permutation is an arrangement in a definite order of a number of objects taken some or all at a time.

\*\* The number of permutations of n different objects taken r at a time, where  $0 < r \leq n$  and the objects do not

repeat is  $n(n - 1)(n - 2) \dots (n - r + 1)$ , which is denoted by

$$P(n, r) \quad \text{OR} \quad {}^n P_r = \frac{n!}{(n - r)!}, 0 \leq r \leq n$$

\*\*  ${}^n P_0 = 1 = {}^n P_n$

\*\* The number of permutations of n different objects taken r at a time, where repetition is allowed, is  $n^r$ .

\*\* The number of permutations of n objects, where  $p_1$  objects are of one kind,  $p_2$  are of second kind, ...,  $p_k$  are of  $k^{\text{th}}$  kind and the rest, if any, are of different kind is  $\frac{n!}{p_1! p_2! \dots p_k!}$ .

\*\* The number of permutations of n dissimilar things taken all at a time along a circle is  $(n - 1)!$ .

\*\* The number of ways of arranging a distinct objects along a circle when clockwise and anticlockwise

arrangements are considered alike is  $\frac{1}{2} (n - 1) !$ .

\*\* The number of ways in which  $(m + n)$  different things can be divided into two groups containing  $m$  and  $n$  things is  $\frac{(m + n) !}{m ! n !}$ .

**Combination** of  $n$  different objects taken  $r$  at a time, denoted by  ${}^n C_r = \frac{n!}{r!(n-r)!}$ .

\*\*  ${}^n P_r = {}^n C_r r!$ ,  $0 \leq r \leq n$

\*\*  ${}^n C_0 = 1 = {}^n C_n$

$${}^n C_1 = n = {}^n C_{n-1}$$

$${}^n C_2 = \frac{n(n-1)}{2!} = {}^n C_2$$

$${}^n C_3 = \frac{n(n-1)(n-2)}{3!} = {}^n C_{n-3}$$

\*\*  ${}^n C_r = {}^n C_s \Rightarrow r = s$  or  $r + s = n$

\*\*  ${}^n C_r + {}^n C_{r-1} = {}^{n+1} C_r$

## QUESTIONS FOR HHW

Q.1 If  ${}^{10}P_r = 5040$ , find the value of  $r$

Q.2 If  ${}^4 C_r = 6 \cdot {}^5 C_{r-1}$ , find value of  $r$ .

Q.3 How many 6-digit number can be formed from the digits 0,1,3,5,7,9 which are divisible by 10

and no digit is repeated ?

Q.4 How many words can be formed by using the letters of the word ORIENTAL, so that the vowels always occupy the odd places ?

Q.5 How many squares in a chess board?

Q.6 How many palindromes of 5 letters can be made by using letters of the word MATHS?

Q.7 It is required to seat 5 men and 4 women in a row so that the women occupy the even places.

How many such arrangements are possible ?

Q.8 Given 12 flags of different colours, how many different signals can be generated if each signal

requires the use of 2 flags, one below the other?

Q.9 There are four bus routes between A and B; and three bus routes between B and C. A man can

travel round-trip in number of ways by bus from A to C via B. If he does not want to use a bus

route more than once, in how many ways can he make round trip?

Q.10 In an examination there are three multiple choice questions and each question has 4 choices.

Find the number of ways in which a student can fail to get all answer correct.

### Long answer type questions

Q.1 A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has (i) no girl ? (ii) at least one boy and one girl ? (iii) at least 3 girls ?

Q.2 Find the number of words with or without meaning which can be made using all the letters of

the word AGAIN. If these words are written as in a dictionary, what will be the 50th word?

Q.3 In an examination, a question paper consists of 12 questions divided into two parts i.e., Part I

and Part II, containing 5 and 7 questions, respectively. A student is required to attempt 8 questions in all, selecting at least 3 from each part. In how many ways can a student select the questions ?

Q.4 How many number of signals that can be sent by 6 flags of different colours taking one or more at a time?

Q.5 A sports team of 11 students is to be constituted, choosing at least 5 from Class XI and at least

5 from Class XII. If there are 20 students in each of these classes, in how many ways can the team be constituted?

## DAY 7(26/10/23)

### **BINOMIAL THEOREM MAIN CONCEPTS AND RESULTS**

\*\* Binomial theorem for any positive integer n

$$(a + b)^n = {}^n C_0 a^n + {}^n C_1 a^{n-1} \cdot b + {}^n C_2 a^{n-2} \cdot b^2 + \dots + {}^n C_{n-1} a \cdot b^{n-1} + {}^n C_n b^n = \sum_{k=0}^n {}^n C_k a^{n-k} b^k$$

\*\* The coefficients  ${}^n C_r$  occurring in the binomial theorem are known as binomial coefficients.

\*\* There are  $(n + 1)$  terms in the expansion of  $(a + b)^n$ , i.e., one more than the index.

$$** (1 + x)^n = {}^n C_0 + {}^n C_1 x + {}^n C_2 x^2 + \dots + {}^n C_{n-1} x^{n-1} + {}^n C_n x^n$$

$$** (1 - x)^n = {}^n C_0 - {}^n C_1 x + {}^n C_2 x^2 - \dots + (-1)^n {}^n C_n x^n .$$

$$** {}^n C_0 + {}^n C_1 + {}^n C_2 + {}^n C_3 + \dots + {}^n C_n = 2^n .$$

$$** {}^n C_0 - {}^n C_1 + {}^n C_2 - {}^n C_3 + \dots + (-1)^{n-1} {}^n C_n = 0 .$$

\*\* General Term in the expansion of  $(a + b)^n = t_{r+1} = {}^n C_r a^{n-r} . b^r$

\*\* Middle term in the expansion of  $(a + b)^n$

- (i)  $\left(\frac{n}{2} + 1\right)^{\text{th}}$  term if n is even      (ii)  $\left(\frac{n+1}{2}\right)^{\text{th}}$  and  $\left(\frac{n+1}{2} + 1\right)^{\text{th}}$  terms if n is odd.

\*\*Pascal's Triangle

Index	Coefficients					
0	${}^0C_0$ (=1)					
1	${}^1C_0$ (=1)				${}^1C_1$ (=1)	
2	${}^2C_0$ (=1)	${}^2C_1$ (=2)			${}^2C_2$ (=1)	
3	${}^3C_0$ (=1)	${}^3C_1$ (=3)	${}^3C_2$ (=3)	${}^3C_3$ (=1)		
4	${}^4C_0$ (=1)	${}^4C_1$ (=4)	${}^4C_2$ (=6)	${}^4C_3$ (=4)	${}^4C_4$ (=1)	
5	${}^5C_0$ (=1)	${}^5C_1$ (=5)	${}^5C_2$ (=10)	${}^5C_3$ (=10)	${}^5C_4$ (=5)	${}^5C_5$ (=1)

### QUESTIONS FOR HHW

<b>1</b>	$(1 + 4x + 4x^2)^{10}$ has 10 TERMS      B) 11 TERMS      C) 20 TERMS      D) 21 TERMS
<b>2</b>	The ratio of the coefficients of $x^r$ and $x^{r-1}$ in $(1 + x)^n$ is A) $\frac{n+r}{r}$ B) $\frac{n-r+1}{r}$ C) $\frac{n+r-1}{r}$ D) NONE
<b>3</b>	If the coefficient of $x^2$ and $x^3$ in the expansion of $(3 + mx)^9$ are equal, then the value of m is A) $-\frac{9}{7}$ B) $-\frac{7}{9}$ C) $\frac{9}{7}$ D) $\frac{7}{9}$
<b>4</b>	The term independent of x in the expansion of $\left(2x + \frac{1}{3x^2}\right)^6$ is A) 2 <sup>nd</sup> B) 3 <sup>rd</sup> C) 4 <sup>th</sup> D) 5 <sup>th</sup>
<b>5</b>	Using binomial theorem, evaluate $(99)^5$ .
<b>6</b>	Expand $\left(x^2 + \frac{3}{x}\right)^4$ , $x \neq 0$ using Pascal triangle.
<b>7</b>	Prove that: $(a + b)^6 - (a - b)^6$ is an even number if a and b are integers:
<b>8</b>	Find 4 <sup>th</sup> term of the expansion $\left(3x + \frac{2}{x}\right)^6$
<b>9</b>	Find 5 <sup>th</sup> term from the end of the expansion $(a + bx)^7$
<b>10</b>	Find $(a + b)^6 - (a - b)^6$ hence evaluate $(\sqrt{3} + \sqrt{2})^6 - (\sqrt{3} - \sqrt{2})^6$
<b>11</b>	Find a, if the 4th and 5th term of the expansion $(2 + a)^7$ are equal.
<b>12</b>	Expand: $(x^2 + 1 - 2x)^3$
<b>13</b>	Find the value of $(a^2 + \sqrt{a^2 - 1})^4 + (a^2 - \sqrt{a^2 - 1})^4$
<b>14</b>	Using binomial theorem prove that $5^{4n} + 52n - 1$ is divisible by 676 $\forall n \in \mathbb{N}$
<b>15</b>	Find the middle term in the expansion $\left[\frac{x}{7} - \frac{5}{x}\right]^6$
<b>16</b>	Find the coefficient of $x^4$ in $\left[2x^2 - \frac{3}{x}\right]^5$
<b>17</b>	Using the binomial theorem, show that $6^n - 5n$ always leaves remainder 1 when divided by 25

18	Find a if coefficients of $x^2$ and $x^3$ in $(3+ax)^9$ are equal.
19	The coefficients of 2 <sup>nd</sup> and 3 <sup>rd</sup> terms in the expansion of $(1+a)^n$ are in the ratio 1:2. Find n.
20	Find the middle term(s) in the expansion of $\left(3x - \frac{x^3}{6}\right)^5$
21	<p><b>Case based Question:</b> In class XI, teacher explained binomial theorem. Two students Shivani and Vishwani trying to solve the exercise. Shivani expanded <math>(1+x)^6</math> by using Binomial theorem, Vishwani expanded <math>(x+1)^6</math>.</p> <p><b>Based on this above information answer the following questions.</b></p> <ol style="list-style-type: none"> <li>1. According to Shivani find 4th term .</li> <li>2. Find the value of <math>{}^6C_0 + {}^6C_1 + {}^6C_2 + {}^6C_3 + \dots + {}^6C_6</math></li> <li>3. Find the positive value of x if 3<sup>rd</sup> terms of Shivani and Vishwani are equal.</li> </ol>

## DAY 8(27/10/23)

### SEQUENCE AND SERIES CONCEPTS AND RESULTS

**\*\* Sequence :** is an arrangement of numbers in a definite order according to some rule. A sequence can also

be defined as a function whose domain is the set of natural numbers or some subsets of the type  $\{1, 2, 3, \dots, k\}$ . **\*\*** A sequence containing a finite number of terms is called a finite sequence. A sequence is called infinite if it is not a finite sequence.

**\*\* Series :** If  $a_1, a_2, a_3, \dots, a_n$ , be a given sequence. Then, the expression  $a_1 + a_2 + a_3 + \dots + a_n + \dots$

**\*\* Arithmetic Progression (A.P.) :** is a sequence in which terms increase or decrease regularly by the same constant.

A sequence  $a_1, a_2, a_3, \dots, a_n, \dots$  is called arithmetic sequence or arithmetic progression if

$a_{n+1} - a_n = d$ ,  $n \in \mathbb{N}$ , where  $a_1$  is called the first term and the constant term  $d$  is called the common difference of the A.P.

**\*\*** The  $n^{\text{th}}$  term (general term) of the A.P.  $a, a+d, a+2d, \dots$  is  **$a_n = a + (n-1)d$** .

**\*\*** If  $a, b, c$  are in A.P. and  $k (\neq 0)$  is any constant, then

(i)  $a+k, b+k, c+k$  are also in A.P.

(ii)  $a-k, b-k, c-k$  are also in A.P.

(iii)  $ak, bk, ck$  are also in A.P.

(iv)  $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$  are also in A.P.

**\*\*** If  $a, a+d, a+2d, \dots, a+(n-1)d$  be an A.P. Then  $l = a + (n-1)d$ .

$$\begin{aligned}\text{Sum to } n \text{ terms } S_n &= \frac{n}{2}[2a + (n-1)d] \\ &= \frac{n}{2}[a + l]\end{aligned}$$

\*\* **Arithmetic mean (A.M.)** between two numbers  $a$  and  $b$  is  $\frac{a+b}{2}$ .

\*\*  **$n$  arithmetic means** between two numbers  $a$  and  $b$  are

$$a + \frac{(b-a)}{n+1}, a + \frac{2(b-a)}{n+1}, a + \frac{3(b-a)}{n+1}, \dots, a + \frac{n(b-a)}{n+1}.$$

\*\* Sum of  $n$  A.M.<sup>s</sup> =  $n$ (single A.M.)

\*\* Three consecutive terms in A.P. are  $a-d, a, a+d$ .

Four consecutive terms in A.P. are  $a-3d, a-d, a+d, a+3d$ .

Five consecutive terms in A.P. are  $a-2d, a-d, a, a+d, a+2d$ .

These results can be used if the sum of the terms is given.

\*\* In an A.P. the sum of terms equidistant from the beginning and end is constant and equal to the sum of first and last terms.

\*\*  $m^{\text{th}}$  term from end of an A.P. =  $(n-m+1)^{\text{th}}$  term from the beginning.

\*\* **Geometric Progression (G.P.)**: A sequence is said to be a geometric progression or G.P., if the ratio of any term to its preceding term is same throughout.

A sequence  $a_1, a_2, a_3, \dots, a_n, \dots$  is called geometric progression, if each term is non-zero and

$$\frac{a_{k+1}}{a_k} = r(\text{constant}), \text{ for } k \geq 1.$$

By taking  $a_1 = a$ , we obtain a geometric progression,  $a, ar, ar^2, ar^3, \dots$ , where  $a$  is called the first term

and  $r$  is called the common ratio of the G.P.

\*\* **General term of a G.P.** =  $a_n = ar^{n-1}$ .

\*\* **Sum to  $n$  terms of a G.P.** =  $\frac{a(r^n - 1)}{r - 1}$  if  $r > 1$  and  $\frac{a(1 - r^n)}{1 - r}$  if  $r < 1$ .

\*\* **Sum of terms of an infinite G.P.** =  $\frac{a}{1 - r}$ .

\*\* **Geometric Mean (G.M.)**: of two positive numbers  $a$  and  $b$  is the number is  $\sqrt{ab}$ .

\*\*  **$n$  geometric mean** between two numbers  $a$  and  $b$  are

$$a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}, a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}, a\left(\frac{b}{a}\right)^{\frac{3}{n+1}}, \dots, a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}.$$

\*\* Three consecutive terms in G.P. are  $\frac{a}{r}, a, ar$ .

Four consecutive terms in G.P. are  $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$ .

Five consecutive terms in G.P. are  $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$ .

These results can be used if the product of the terms is given

## QUESTIONS FOR HHW

Q1. If  $3^{\text{rd}}$ ,  $8^{\text{th}}$  and  $13^{\text{th}}$  terms of G.P. are  $p, q$  and  $r$  respectively, then which one of the following is correct

- a.  $q^2 = pr$       b.  $r^2 = pq$       c.  $pq = r$       d.  $2q = p+r$

Q2. If  $n^{\text{th}}$  term of a sequence is  $a_n = (-1)^{n-1} n^3$ , then its  $9^{\text{th}}$  term is

- A) 105      B) 177      C) 324      D) 729

Q3. geometric mean between 1 and 256 is

- A) 8      b. 16      c. 14      d. 12

Q4. If  $x, 2x+3, 3x+3$  are in G.P., then  $4^{\text{th}}$  term is

- A) -13.5      b. -14.5      c. -15.5      d. -16.5

Q5. Statement I: Four terms of the G.P.  $3, 3^2, 3^3, \dots$  are needed to give the sum 120

Statement II:  $T_n = ar^n$  is  $n^{\text{th}}$  terms of G.P. whose first term is  $a$  and common ratio  $r$ .

- Both the statement I and Statement II are true and statement II is the correct explanation of Statement I
- Both the statement I and Statement II are true and statement II is not the correct explanation of Statement I
- Statement I is true but Statement II is false
- Statement I is false but Statement II is true

Q6 Write first five terms of sequence whose  $n^{\text{th}}$  term is given by  $a_n = (-1)^{n-1} 5^{n+1}$

Q7. for what values of  $x$ , the numbers  $-2/7, x, -7/2$  are in G.P.

Q8. Find the  $12^{\text{th}}$  term of a G.P. whose  $8^{\text{th}}$  term is 192 and the common ratio is 2.

Q9. The sum of first three terms of a G.P. is  $\frac{39}{10}$  and their product is 1. Find the common ratio

and

the terms.

Q10. Find the sum to  $n$  terms of the sequence, 8, 88, 888, 8888...

11. Let  $S$  be the sum,  $P$  the product and  $R$  the sum of reciprocals of  $n$  terms in a G.P. Prove that

$$P^2 R^n = S^n.$$

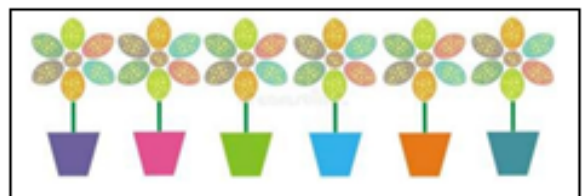
12. If  $A$  and  $G$  be A.M. and G.M., respectively between two positive numbers, prove that the numbers are  $A \pm \sqrt{(A+G)(A-G)}$ .

### 13 Case study

Rahul being a plant lover decides to open a nursery and he bought few plants with pots. He wants

to place pots in such a way that number of pots in first row is 2, in second row is 4 and in third row is 8 and so on. Answer the following questions based on the above information.

- Find the number of pots in the 8th row.
- Find the total number of pots in 10 rows.
- If Rahul wants to place 510 pots in all, how many rows will be formed?





# DAY 9(28/10/23)

## STRAIGHT LINES CONCEPTS AND RESULTS

\*\* Any point on the X-axis is  $(x, 0)$  and on the Y-axis is  $(0, y)$

\*\* Distance between two points  $A(x_1, y_1)$  &  $B(x_2, y_2)$  is  $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ .

### \*\* Section formula

(i) Coordinates of a point dividing the line joining  $A(x_1, y_1)$  &  $B(x_2, y_2)$  internally in the ratio  $m : n$  is

$$\left( \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right).$$

(ii) Coordinates of a point dividing the line joining  $A(x_1, y_1)$  &  $B(x_2, y_2)$  externally in the ratio  $m : n$  is

$$\left( \frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n} \right).$$

\*\* Coordinates of the mid point of the line joining  $A(x_1, y_1)$  &  $B(x_2, y_2)$  is

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

\*\* Centroid of a  $\Delta ABC$  with vertices  $A(x_1, y_1)$ ,  $B(x_2, y_2)$  &  $C(x_3, y_3)$

$$\left( \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right).$$

\*\* In centre of  $\Delta ABC$  with vertices  $A(x_1, y_1)$ ,  $B(x_2, y_2)$  &  $C(x_3, y_3)$  is

$$\left( \frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right) \text{ where } a = BC, b = AC, c = AB.$$

\*\* Area of  $\Delta ABC$  with vertices  $A(x_1, y_1)$ ,  $B(x_2, y_2)$  &  $C(x_3, y_3) = \frac{1}{2} | x_1(y_2 - y_3) + x_2(y_3 - y_1)$

$+ x_3(y_1 - y_2) |$ .

\*\* Equation of any line parallel to X-axis is  $y = a$ , & equation of X-axis is  $y = 0$ .

\*\* Equation of any line parallel to Y-axis is  $x = b$  & equation of Y axis is  $x = 0$ .

\*\* Slope of line inclined at an angle  $\theta$  with the +ve X-axis =  $\tan \theta$ .

\*\* Slope of a line parallel to X-axis = 0, slope of a line parallel to Y-axis = undefined.

Slope of a line equally inclined to the coordinate axes is  $-1$  or  $1$ .

\*\* Slope of a line joining the points  $A(x_1, y_1)$ ,  $B(x_2, y_2)$  is  $\frac{y_2 - y_1}{x_2 - x_1}$ ,  $x_1 \neq x_2$ .

\*\* Slope of the line  $ax + by + c = 0$ , is  $-\frac{a}{b}$ .

\*\* If two lines are parallel, then their slopes are equal.

\*\* If two lines are perpendicular, then the product of their slopes is  $-1$ .

\*\* Any equation of the form  $Ax + By + C = 0$ , with A and B are not zero, simultaneously, is called the general linear equation or general equation of a line.

(i) If  $A = 0$ , the line is parallel to the x-axis (ii) If  $B = 0$ , the line is parallel to the y-axis

(iii) If  $C = 0$ , the line passes through origin.

\*\* Equation of a line having slope = m and cutting off an intercept 'c' and Y-axis is  $y = mx + c$ .

\*\* Equation of a line through the point  $(x_1, y_1)$  and having slope m is  $y - y_1 = m(x - x_1)$ .

\*\* Equation of a line making intercepts of 'a' & 'b' on the respective axes is  $\frac{x}{a} + \frac{y}{b} = 1$

\*\* The equation of the line having normal distance from origin p and angle between normal and the

positive x-axis  $\omega$  is given by  $x \cos\omega + y \sin\omega = p$ .

\*\* Distance of a point  $P(x_1, y_1)$  from the line  $ax + by + c = 0$  is  $d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$ .

\*\* Equation of the line parallel to  $ax + by + c = 0$  is  $ax + by + \lambda = 0$ .

\*\* Equation of the line perpendicular to  $ax + by + c = 0$  is  $bx - ay + \lambda = 0$ .

\*\* If two lines are intersecting and  $\theta$  is the angle between them, then  $\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|$

where  $m_1 =$  slope of first line,  $m_2 =$  slope of second line and  $\theta =$  acute angle.

If  $\tan \theta =$  negative  $\Rightarrow \theta =$  obtuse angle between the intersecting lines.

\*\* Distance between two parallel lines  $ax + by + c_1 = 0$  &  $ax + by + c_2 = 0$  is  $\left| \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \right|$ .

## QUESTIONS FOR HHW

1) Slope of a line which cuts off intercepts of equal lengths on the axes is

- (a) -1 (b) 0 (c) 2 (d)  $\sqrt{3}$

2. The value of y so that the line through  $(3, y)$  and  $(2, 7)$  is parallel to the line through  $(-1, 4)$  and  $(0, 6)$  is

- a) 7 (b) 10 (c) 9 (d) 8

3) The radius of the circle  $x^2 + y^2 + 8x + 10y - 8 = 0$  is

- a) 8 (b) 10 (c) 9 (d) 7

4) The focus of the parabola  $y^2 = -8x$  is

- A)  $(2, 0)$  b)  $(-2, 0)$  c)  $(0, 2)$  d)  $(0, -2)$

5) Assertion (A). The slope of a line passing through two points  $(-5, 2)$  and  $(3, -2)$  is  $-\frac{1}{2}$

Reason (R). The slope of a line passing through two given points (  $x_1, y_1$  ) and (  $x_2, y_2$  ) is  $\frac{x_2-x_1}{y_2-y_1}$

- a) Both A and R are true and R is the correct explanation of A.
- b) Both A and R are true but R is not correct explanation of A.
- c) A is true but R is false
- d) A is false but R is true.  
Both A and R are false.
- e)